Penrith Selective High School

2017

Trial Higher School Certificate
Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A separate reference sheet is to be provided for this examination paper
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- All diagrams are not to scale
- Multiple choice answer sheet is on page 17 of this paper

Total Marks – 100

Section I Pages 3 – 6

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 7 – 16

90 marks

- Attempt Questions 11–16
- Allow about 2 hour 45 minutes for this section

Student Number: ____

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2017 Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet provided on page 17 for Questions 1–10.

Q1. What is the derivative of $\frac{3}{x}$?

(A) $\frac{3}{x^2}$

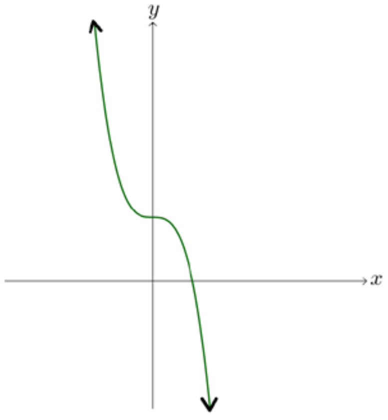
(B) $-\frac{3}{x^2}$

(C) $-3x$

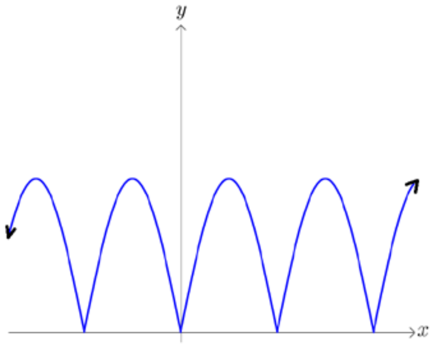
(D) $-\frac{6}{x^2}$

Q2. Which of the following is an EVEN function?

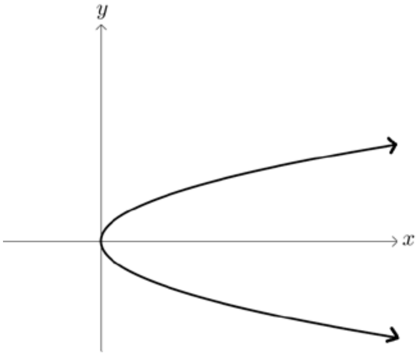
(A)



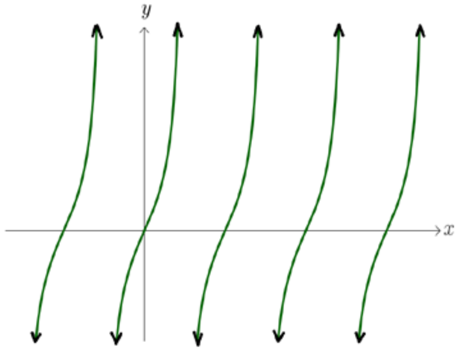
(B)



(C)



(D)



Q3. Which statement is FALSE?

(A) A trapezium is a quadrilateral with one pair of opposite sides parallel.

(B) The diagonals of a rectangle are perpendicular.

(C) A kite can be divided into two congruent triangles.

(D) The properties of a rectangle has the same properties as a parallelogram.

Q4. Find the domain of $y = \frac{1}{\sqrt{6 - x}}$

(A) $x > 6$

(B) $x \geq 6$

(C) $x < 6$

(D) $x \leq 6$

Q5. What is the nature of the roots of the quadratic equation $x^2 - 8x - 48 = 0$?

(A) Real, rational and equal

(B) Real, irrational and unequal

(C) Real, rational and unequal

(D) Unreal, irrational and unequal

Q6. Find the primitive function of $2e^{3x} - 4x$.

(A) $6e^{3x} - 4x^2 + C$

(B) $\frac{2}{3}e^{3x} - 4x + C$

(C) $\frac{2}{3}e^{3x} - 2x^2 + C$

(D) $2e^{3x} - 4x^2 + C$

Q7. A function $y = f(x)$ has $f'(4) = 0$ and $f''(4) = -2$.

At the point where $x = 4$, $y = f(x)$ is:

(A) Stationary and concave up.

(B) Decreasing and concave down.

(C) Stationary and concave down.

(D) Stationary with a horizontal point of inflexion.

Q8. What is the equation of the directrix of the parabola $y^2 = -12(x - 5)$?

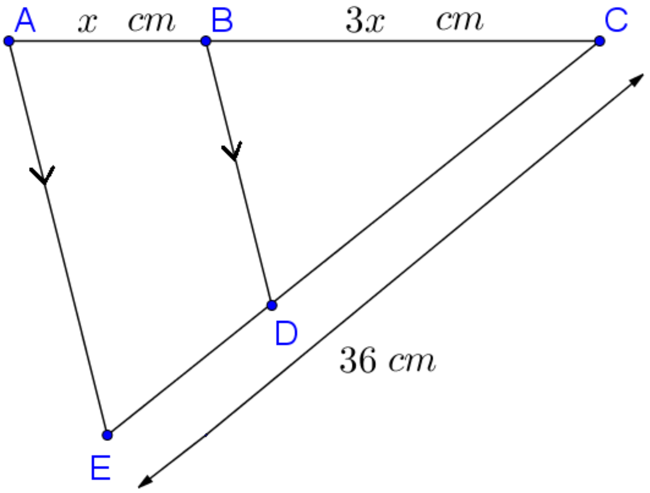
(A) $y = -2$

(B) $y = 8$

(C) $x = 2$

(D) $x = 8$

Q9. In the diagram AE is parallel to BD, $AB = x$ cm, $BC = 3x$ cm and $EC = 36$ cm.



The length of DC is:

(A) 6 cm

(B) 9 cm

(C) 18 cm

(D) 27 cm

Q10. Two bags each contain blue marbles and green marbles. Bag A contains 4 blue and 4 green marbles. Bag B contains 2 blue and 3 green marbles. A marble is randomly chosen from each bag. The probability that both marbles are of the same colour is?

(A) $\frac{2}{5}$

(B) $\frac{3}{5}$

(C) $\frac{1}{2}$

(D) $\frac{4}{5}$

END OF SECTION I

Section II

90 Marks
Attempt Questions 11–16
Allow about 2 hour and 45 minutes for this section

Answer each question on a SEPARATE booklet.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

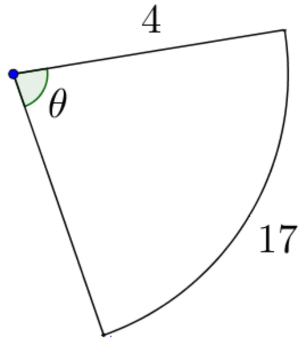
Question 11 (15 marks) **Start this question on a new writing booklet**

a) Evaluate $\dfrac{\sqrt{7^2 + 196}}{13 - 8}$ to three significant figures. **2**

b) Solve $|2x + 3| < 21$ **2**

c) Differentiate $\left(3 + \dfrac{x^2}{5}\right)^5$ **2**

d) An arc length of 17 units subtends an angle θ at the centre of the circle with radius 4 units as shown below. **2**



Find the area of the sector shown above.

e) State the coordinate of the centre and the radius of the circle given by $x^2 + y^2 - 6x + 12y - 124 = 0$. **3**

Question 11 continues on page 8

Question 11 continued

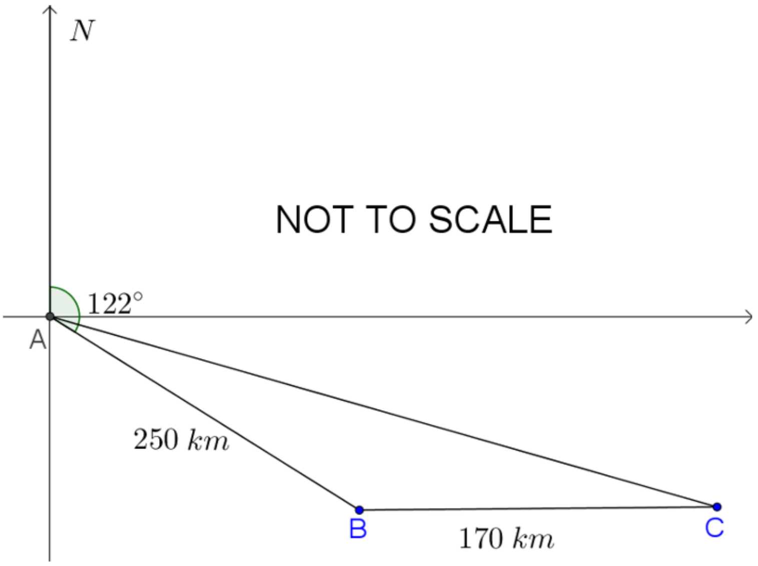
f) Find $\int \frac{5x}{x^2 - 9}\,dx$ **2**

g) Solve for x: $3^{5-2x} = \frac{1}{\sqrt[3]{27}}$ **2**

End of Question 11

Question 12 (15 marks) **Start this question on a new writing booklet**

a) The diagram below represents the journey taken by a ship which leaves point A and travels $250\ km$ on a bearing of $122°$ to B. It then turns and travels $170\ km$ due east to C.



i) Show that $\angle ABC = 148°$ **1**

ii) Find the distance AC correct to the nearest km. **1**

iii) Find the bearing of A from C. Round your answer to the nearest degree. **2**

b) If α and β are the roots of $3x^2 + 4x - 3 = 0$. Find the values of:

i) $\alpha + \beta$ **1**

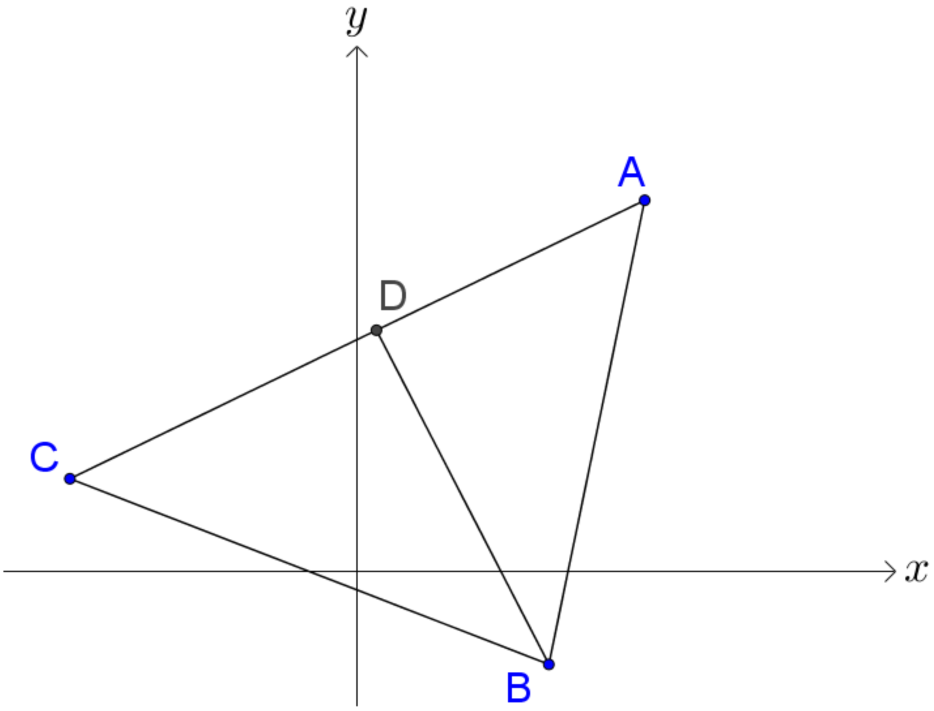
ii) $2\alpha^2 + 2\beta^2$ **2**

c) Find the equation of the tangent to the curve $y = 3xe^{-x}$ at the point $\left(2, \frac{6}{e^2}\right)$. **2**

Question 12 continues on page 10

Question 12 continued

d) The coordinates of the points $A\ (15, 20)$, $B\ (10, -5)$ and $C(-15\ , 5)$, are shown in the diagram. Point $D(1, 13)$ lies on the line passing through A and C .



i) Show that the equation of the interval AC is $x - 2y + 25 = 0$. **2**

ii) Find the exact length of AC. **1**

iii) Show that BD is perpendicular to AC. **1**

iv) Hence, find the exact length of BD. **1**

v) Hence or otherwise, find the area of ΔABC. **1**

End of Question 12

Question 13 (15 marks) **Start this question on a new writing booklet**

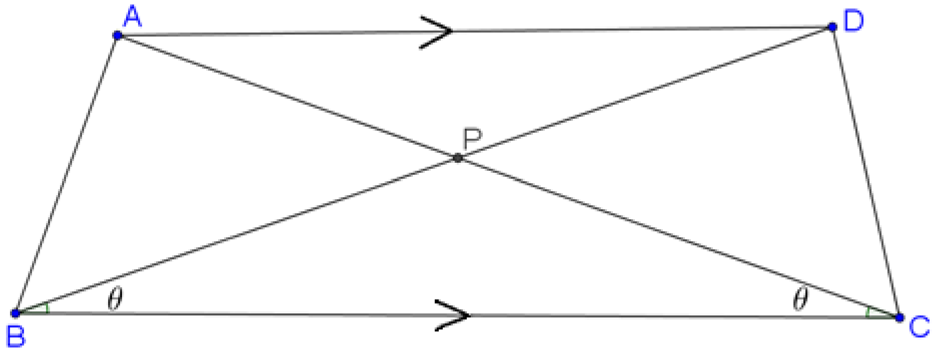
a) Differentiate with respect to x

i) $x \cos 5x$ **2**

ii) $\dfrac{\ln x}{x^2}$ **2**

b) The first term of an arithmetic progression is 5, and the ninth term is three times the fourth term. What is the value of the common difference? **2**

c) In the diagram shown below, AD is parallel to BC. $\angle DBC = \angle ACB = \theta$



i) Show that $AP = PD$ **2**

ii) Prove that $\Delta APB \equiv \Delta DPC$ **2**

d) Given that $\int_0^6 (kx - 5)\ dx = 12$ and k is a constant, find the value of k. **2**

e) Show that $\sqrt{\dfrac{1 - sin^2\theta}{cosec^2\theta - cot^2\theta - cos^2\theta}} = cot\theta$ **3**

End of Question 13

Question 14 (15 marks) **Start this question on a new writing booklet**

a) Let $\log_a 3 = x$ and $\log_a 5 = y$.

Find an expression in terms of x and y for

i) $\log_a 0.6$ **1**

ii) $\log_a 45a$ **2**

b) Kelly and Patrick compete in a series of games. The series finishes when one player has won two games. In any game, the probability that Kelly wins is $\frac{2}{5}$ and the probability that Patrick wins is $\frac{3}{5}$.

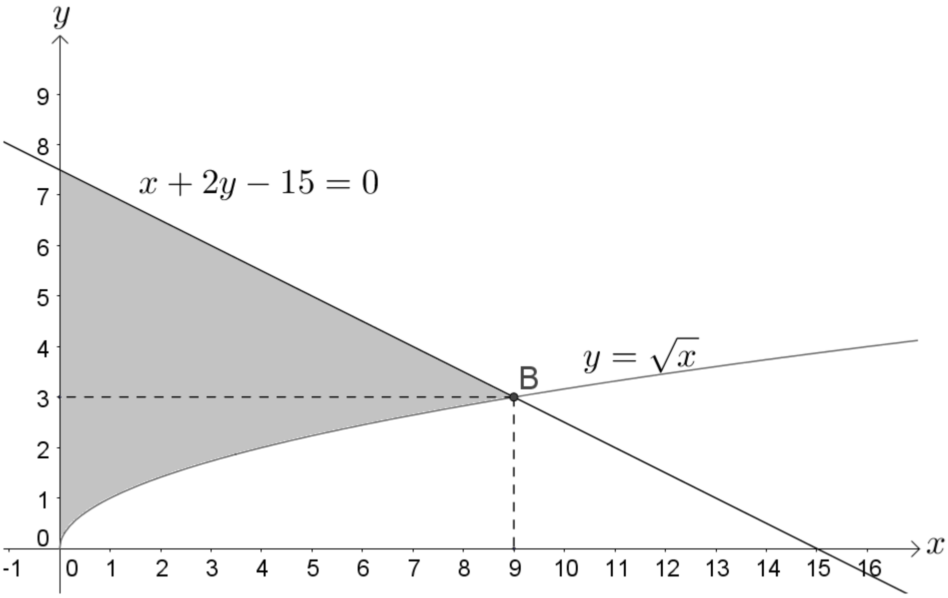
i) What is the probability that Patrick wins the series? **2**

ii) What is the probability that three games are played in the series? **2**

c) For what values of x will the following geometric series have a limiting sum? **2**

$$1 + (4 - x) + (4 - x)^2 + \ldots\ldots$$

d) Find the shaded area in the diagram below. **3**



Question 14 continues on page 13

e) Consider the function $f(x) = |x - 6|$

i) Sketch $f(x)$, showing all key features. **1**

ii) Hence or otherwise, evaluate $\int_0^8 |x - 6| \, dx$ **2**

End of Question 14

Question 15 (15 marks) **Start this question on a new writing booklet**

a) Find the solutions of $\sqrt{3}\tan 2x = 1$ for $0 \leq x \leq 2\pi$ **3**

b) Use Simpson's rule with 3 function values to find an approximation for **2**

$\int_1^5 x \ln x \, dx$, correct to two decimal places.

c) The region bounded by the curve $y = \sec 2x$ the lines $x = \frac{\pi}{8}$ **3**

and $x = \frac{\pi}{6}$ is rotated about the x axis. Find the volume of solid of revolution. Give your answer in exact form.

d) Consider the function $y = 3\cos 2x$

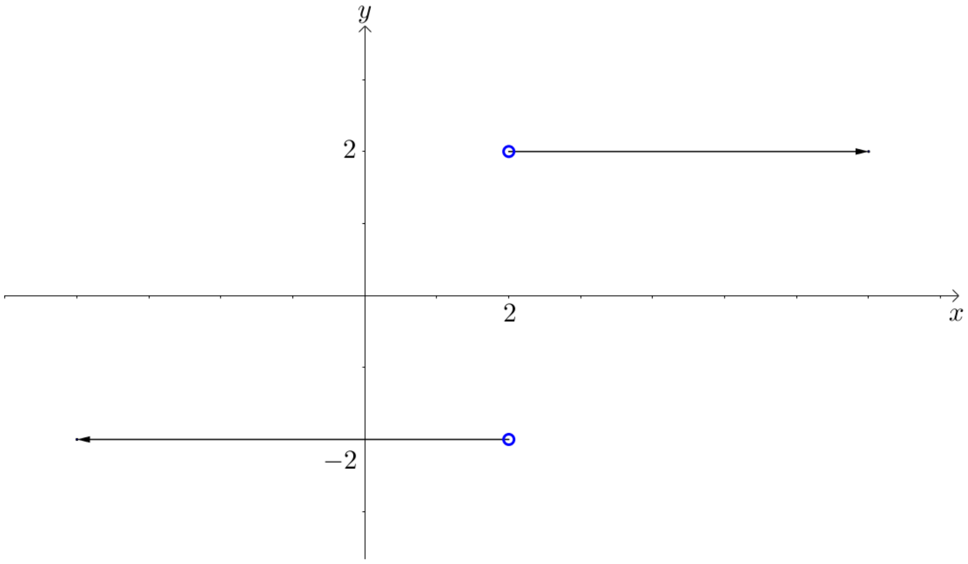
i) Write down the amplitude and period of this function. **2**

ii) Sketch the curve for $0 \leq x \leq \pi$. Showing all intercepts. **2**

iii) Find the area bounded be the curve, the x axis, $x = 0$ and $x = \frac{\pi}{2}$. **1**

e) Sketch a possible function which could have the gradient function **2**

as graphed below.



End of Question 15

Question 16 (15 marks) **Start this question on a new writing booklet**

a) i) Show $\dfrac{6x+4}{2x+1} = 3 + \dfrac{1}{2x+1}$ **1**

ii) Hence find $\displaystyle\int \frac{6x+4}{2x+1}\,dx$ **2**

b) At the completing of her degree, Manpreet had a Higher Educational Loan Payment (HELP) debt of $70 000. She plans to repay this in equal monthly repayments of $\$M$. Interest is charged at a rate of 0.4% per month.

Let $\$A_n$ be the amount owing at the end of the nth month.

i) Show that the amount owing after 3 months is given by **1**

$$A_3 = 70\ 000 \times 1.004^3 - M(1 + 1.004 + 1.004^2)$$

ii) If Manpreet decides that she would like to pay off her loan by the end of ten years, how much would her monthly repayment be? correct to the nearest cent. **2**

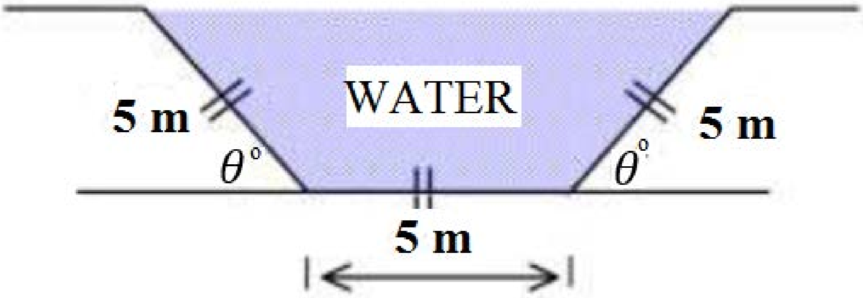
iii) If Manpreet decides that she can only repay $450 each month, how long will it take her to repay the loan? **2**
(Answer in years and months)

Question 16 continues on page 16

Question 16 continued

c) An irrigation channel has a cross-section in the shape of a trapezium as shown in the diagram. The bottom and sides of the trapezium are 5 metres long.

Suppose that the sides of the channel make an angle θ with the horizontal where $\theta \leq \frac{\pi}{2}$.



i) Show that the cross-sectional area is given by $A = 25(\sin\theta + \sin\theta\cos\theta)$ **2**

ii) Show that $\dfrac{dA}{d\theta} = 25(2cos^2\theta + \cos\theta - 1)$ **2**

iii) Hence, show that the maximum cross-sectional area occurs when $\theta = \dfrac{\pi}{3}$ **2**

iv) Hence, find the maximum area of the irrigation channel correct to the nearest square metre. **1**

End of Paper

Student Number:____

Multiple Choice Answer Sheet

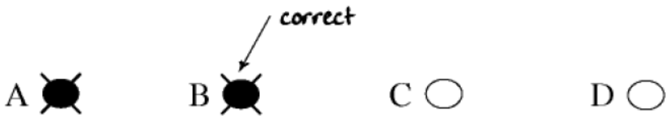
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ‘correct’ and drawing an arrow as follows.



Start Here ➔

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

9. A ○ B ○ C ○ D ○

10. A ○ B ○ C ○ D ○

Exam 2U Mathematics MATHEMATICS : Question......11

Suggested Solutions	Marker's Comments
Q11 a) $\dfrac{\sqrt{7^2+196}}{13-8}$ $= 3.130495168\ldots$ $= 3.13$ (3 sig. fig)	Most students answered this correctly. A few students left their answer as 3.130 which is 4 sig. fig.
b) $\|2x+3\| < 21$ $2x+3 < 21$ $-(2x+3) < 21$ $2x < 18$ $-2x-3 < 21$ $x < 9$ $-2x < 24$ $x > -12$ $\therefore -12 < x < 9$	some students left it as $x<9$ or $x > -12$ which is incorrect.
c) $\left(3+\dfrac{x^2}{5}\right)^5$ $= 5\left(3+\dfrac{x^2}{5}\right) \times \dfrac{2x}{5}$ $= 2x\left(3+\dfrac{x^2}{5}\right)$	
Multiple Choice Answers 1. B 2. B 3. B 4. C 5. C 6. C 7. C 8. D 9. D 10. C	

Exam 2u Mathematics MATHEMATICS : Question 1.1..

Suggested Solutions	Marker's Comments
d) $l = r\theta$ $17 = 4\theta$ $\theta = \frac{17}{4}$ $A = \frac{1}{2}\theta r^2$ $= \frac{1}{2} \times \frac{17}{4} \times 4^2$ $= 34$ units^2	Some students had $\theta = \frac{4}{17}$ or they try to convert to $\frac{17\pi}{4}$ which is incorrect.
e) $x^2 + y^2 - 6x + 12y - 124 = 0$ $x^2 - 6x + 9 + y^2 + 12y + 36 = 124 + 9 + 36$ $(x-3)^2 + (y+6)^2 = 169$ Circle centre $(3, -6)$ radius 13	Most students who knew how to complete the square, were successful in obtaining the circle centre and the radius
f) $\int \frac{5x}{x^2-9}\,dx$ $= \frac{5}{2}\int \frac{2x}{x^2-9}\,dx$ $= \frac{5}{2}\ln\lvert x^2-9\rvert + C$ g) $3^{5-2x} = \frac{1}{\sqrt[3]{27}}$ $3^{5-2x} = 3^{-1}$ $5 - 2x = -1$ $2x = -6$ $x = -3$	Common error some students had $\frac{1}{\sqrt[3]{27}}$ as 3 rather than 3^{-1}

Suggested Solutions | Marker's Comments

12a i $\angle EAB = 32^\circ \; (122^\circ - 90^\circ)$

$\therefore \angle EAB + \angle ABC = 180^\circ$

$\angle ABC = 180 - 32^\circ$ ✓

$= 148^\circ$

ii $AC^2 = 250^2 + 170^2 - 2 \times 250 \times 170 \times \cos 148^\circ$

$= 163{,}484.08\ldots$

$AC = 404.33$ ✓

$= 404$

iii) $\dfrac{\sin C}{250} = \dfrac{\sin B}{404.33}$

$\sin C = \dfrac{250 \sin 148}{404.37}$ ✓

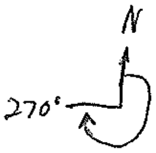
$\therefore C = 19^\circ$

Students Did not use the sine rule

$\therefore$ Bearing of A from C

$= 270 + 19$ ✓

$= 289^\circ$



Students did not add their answer above to 270°

b i) $\alpha + \beta = \dfrac{-b}{a}$ ✓

$\alpha + \beta = -\dfrac{4}{3}$

ii) $2(\alpha^2 + \beta^2)$

$= 2\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$

$= 2\left[\left(\frac{4}{3}\right)^2 - 2 \times 1\right]$ ✓ ✓

$= 2\left[\frac{16}{9} + 2\right]$

$= 68/9$

Student did not know $(a+b)^2 - 2ab$

Suggested Solutions	Marker's Comments

c) $y = 3x \times e^{-x}$

$y' = 3x \times e^{-x} \times -1 + e^{-x} \times 3$

$= -3xe^{-x} + 3e^{-x}$

$= -3e^{-x}(x - 1)$

$f'(2) = -3e^{-2}$

Some student failed to sub in $x = 2$

$y - y_1 = m(x - x_1)$

$y - \frac{6}{e^2} = -3e^{-2}(x - 2)$ ✓

Some student did not multiply by e^2 at this line

$e^2 y - 6 = -3x + 6$

$e^2 y = -3x + 12$ ①

d i $m = \frac{15}{30}$

$= \frac{1}{2}$

$y - 20 = \frac{1}{2}(x - 15)$

$y = \frac{1}{2}x - \frac{15}{2} + 20$

$y = \frac{x}{2} + 12\frac{1}{2}$

$2y = x + 25$ ②

ii) $d = \sqrt{30^2 + 15^2}$

$= \sqrt{1125}$

$= 15\sqrt{5}$

iii) $m(BD) = \frac{18}{-9}$

$= -2$

Suggested Solutions | Marker's Comments

(iii) Continued

$m(AC) = \frac{15}{30}$

$= \frac{1}{2}$

$\therefore BD \perp AC$ Since $m(BD) \times m(AC) = -2 \times \frac{1}{2} = -1$ ①

(iv) $d = \sqrt{18^2 + 9^2}$

$= \sqrt{405}$

$= 9\sqrt{5}$ ①

Some students using ⊥ distance formula to a straight line. Unnecessary

(v) $A = \frac{1}{2} \times 15\sqrt{5} \times 9\sqrt{5}$

$= 337.5 \text{ units}^2$ ①

Some student still had irrational answer.

Exam Trial HSC 2017 MATHEMATICS : Question 13.

Suggested Solutions | Marker's Comments

a) i) $y = x\cos 5x$

$$\frac{dy}{dx} = 1\cos 5x + x \times -5\sin 5x$$

(ii) $y = \frac{\ln x}{x^2}$

$$\frac{dy}{dx} = \frac{x^2 \times \frac{1}{x} - \ln x \times 2x}{x^4}$$

$$= \frac{x - 2x\ln x}{x^4} \text{ or } \boxed{\frac{1-2\ln x}{x^3}}$$

- 2 marks each Suggesting some working could be good.

b) $a = 5$ $T_9 = a + 8d$

$T_4 = a + 3d$

$a + 8d = 3(a + 3d)$

$a + 8d = 3a + 9d$

$-2a = d$

$d = -10$

- Some students put this 3 on the left hand side instead.

$x = y$
$p = q$
Does not mean $x = q$.

- Not correct If you do not put $= \theta$ or otherwise say $\angle DBC = \angle ACB$ given.

c) i) $\angle DAP = \angle ACB = \theta$ (alternate angles AD ∥ BC, given)

$\angle ADP = \angle DBC = \theta$ (as above)

$\therefore \angle DAP = \angle ADP$

$\therefore AP = PD$ (equal sides opposite equal angles of $\triangle DAP$)

This explanation was often muddled by students.

- Stating AD∥BC is essential. It is better than writing 'Parallel lines'. Be specific. Although parallel lines was accepted on this occasion.
- Do not talk about corresponding sides of isosceles triangles.

(ii) In $\triangle APB$, $\triangle DPC$

$BP = PC$ (equal sides opposite equal angles in $\triangle BPC$, given $\angle DBC = \angle ACB = \theta$)

$\angle APB = \angle DPC$ (Vertically opposite)

- Make P look different to D
- Make letter C look different to left hand parenthesis (

Suggested Solutions	Marker's Comments
C(ii) cont. $AP = PD$ (from(i)) $\therefore \underline{\triangle APB \equiv \triangle DPC}$ (SAS)	• Proof of equiangular triangles is not a proof of congruency! (It is a proof of similarity).
d) $\int_0^6 (kx-5)\,dx = 12$ $= \left[\frac{kx^2}{2} - 5x\right]_0^6$ $= K \times 18 - 30 - 0 = 12$ $18K = 42$ $\underline{K = 2\frac{1}{3}}$	This was very well answered.
e) LHS $= \sqrt{\frac{1 - \sin^2\theta}{\operatorname{cosec}^2\theta - \cot^2\theta - \cos^2\theta}}$ $= \sqrt{\frac{\cos^2\theta}{1+\cancel{\cot^2\theta}-\cancel{\cot^2\theta}-\cos^2\theta}}$ $(\operatorname{cosec}^2\theta = 1 + \cot^2\theta)$ $= \sqrt{\frac{\cos^2\theta}{1-\cos^2\theta}}$ $= \sqrt{\frac{\cos^2\theta}{\sin^2\theta}}$ $= \sqrt{\cot^2\theta}$ $= \cot\theta$ $=$ RHS	• Generally well answered but • too many students square rooted each term seperately and said $=$ $\frac{1-\sin\theta}{\operatorname{cosec}\theta-\cot\theta-\cos\theta}$ $\sqrt{a^2+b^2}$ $\neq a+b$ Take care!

Exam Mathematics 2 Unit. MATHEMATICS : Question..17

Suggested Solutions	Marker's Comments
a) i) $\log_a 0.6$ $= \log_a\left(\frac{3}{5}\right)$ $= \log_a 3 - \log_a 5$ $= x - y$	Most students answered this correctly.
ii) $\log_a 45a$ $= \log_a(3^2 \times 5 \times a)$ $= \log_a 3^2 + \log_a 5 + \log_a a$ $= 2\log_a 3 + \log_a 5 + \log_a a$ $= 2x + y + 1$	Most students did this well but were not sure about $\log_a a$
b) $P(\text{Patrick wins}) = \frac{3}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5} + \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5}$ $= \frac{81}{125}$	Students did not do this well.
ii) Complement of KK + PP $1 - \left(\left(\frac{2}{5}\right)^2 + \left(\frac{3}{5}\right)^2\right)$ $= \frac{12}{25}$	Most students did not do this question well.

c) $|r| < 1$ for limiting sum

$r = 4 - x$

$-1 < 4 - x < 1$

$-5 < -x < -3$

$5 > x > 3$

$3 < x < 5$

Students incorrectly used $S_\infty = \frac{a}{1-r}$

d) $\int_0^3 y^2 \, dy + \int_3^{7.5} (15 - 2y) \, dy$

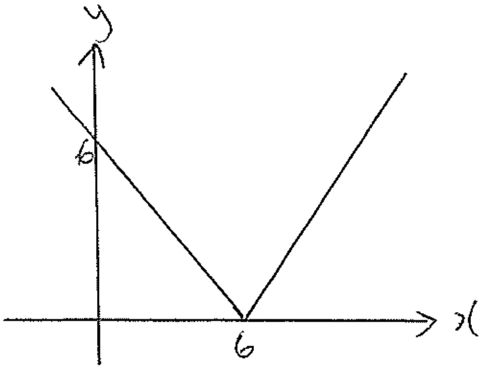
$= \left.\frac{y^3}{3}\right|_0^3 + \left.15y - \frac{2y^2}{2}\right|_3^{7.5}$

$= 9 + 56.25 - (45 - 9)$

$= 9 + 20.25$

Most students did this well. Many methods

e)



f) $\int_0^8 |x - 6| \, dx$

$= \frac{1}{2} \times 6 \times 6 + \frac{1}{2} \times 2 \times 2$

$= 18 + 2$

$= 20$

Students Over

Exam YR 12 2UNIT TRIAL MATHEMATICS : Question...... 15

Suggested Solutions | **Marker's Comments**

a. $\sqrt{3}\tan 2x = 1$

$\tan 2x = \frac{1}{\sqrt{3}}$ $\qquad 0 \le x \le 2\pi$

$2x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}$ $\qquad 0 \le 2x \le 4\pi$

$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$

must answer in radians $0 \le x \le 2\pi$!

b. $\int_1^5 \doteqdot \frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right)$

$= \frac{5-1}{6}\left(1\ln 1 + 4(3\ln 3) + 5\ln 5\right)$

$= \frac{4}{6}\left(0 + 12\ln 3 + 5\ln 5\right)$

$= 14.15$ (2 dec. pl.)

Simple question badly answered

look at Reference Sheet for formula Suggested.

c. $V = \pi\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \sec^2 2x\,dx = \pi\left[\frac{1}{2}\tan 2x\right]_{\frac{\pi}{8}}^{\frac{\pi}{6}}$

$=$

$= \pi\left(\frac{1}{2}\tan\frac{\pi}{3} - \frac{1}{2}\tan\frac{\pi}{4}\right)$

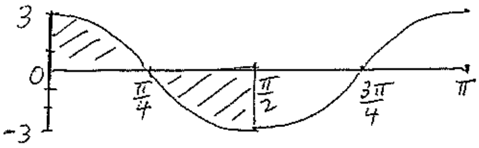
$= \pi\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$

$= \frac{\pi}{2}(\sqrt{3} - 1)\ u^3.$

$\int \sec^2 x\,dx = \tan x + c$

d. Amplitude 3

Period $\frac{2\pi}{2} = \pi$



$\text{Area} = 2\int_0^{\frac{\pi}{4}} 3\cos 2x\,dx$

$= 2\left[\frac{3}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$

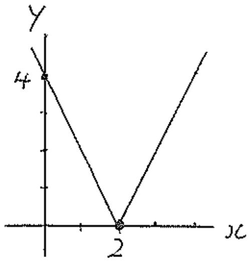
$= 3\sin 2\left(\frac{\pi}{4}\right) - 0$

$= 3\sin\frac{\pi}{2}$

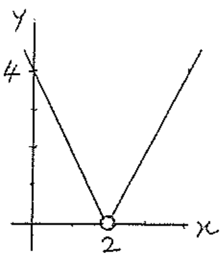
$= 3\ u^2$

Area between curve and x-axis is NOT zero. but $2\int_0^{\frac{\pi}{4}}$ by Symmetry.

e.



or



gradients must be Clearly shown as 2 and −2

both graphs are non-differentiable at x = 2.

Q16 Solutions 20 /7

's Comments

16/

a) (i) $\frac{6x+4}{2x+1} = 3 + \frac{1}{2x+1}$.

$$RHS = \frac{(6x+3)+1}{2x+1}$$

$$= \frac{6x+3}{2x+1} + \frac{1}{2x+1}$$

$$= 3 + \frac{1}{2x+1}$$

/1

MINUS 1 MARK FOR ea MISTAKE

(ii) $$\int \frac{6x+4}{2x+1}\,dx = \int 3 + \frac{1}{2x+1} = \int 3dx + \int \frac{1}{2x+1}dx$$

$$= 3x + \frac{1}{2}\int \frac{2}{2x+1}dx$$

$$= 3x + \frac{1}{2}\ln|(2x+1)| + C$$

/2

MARKS MINUS 1 FOR EACH MISTAKE

b) (i) Amount owing 1 month

$$A_1 = (70000 \times 1.004) - M$$
$$A_2 = (A_1 \times 1.004) - M$$
$$A_3 = (A_2 \times 1.004) - M$$

PATTERN IMPORTANT

$$\therefore A_2 = A_1 \times 1.004 - M = ([70000 \times 1.004) - M] \times 1.004) - M$$

$$= 70000 \times 1.004^2 - M(1 + 1.004)$$

$$A_3 = (A_2 \times 1.004) - M = [70000 \times 1.004^2 - M(1+1.004)] \times 1.004 - M$$

$$= 1.004^3 \times 70000 - M(1 + 1.004 + 1.004^2)$$

/1

(ii) If $A_{120} = 0$

$$\therefore 0 = 1.004^{120} \times 70000 - M(1 + 1.004 + 1.004^2 + \ldots 1.004^{119})$$

$$M = \frac{1.004^{120} \times 70000}{S_{120}}$$

Sum of GP since $r = 1.004$. $S_{120} = \frac{a(r^n - 1)}{r-1}$

$$= \frac{1(1.004^{120} - 1)}{0.004}$$

(Cont)

$$M = \frac{1.004^{120} \times 70000}{(1.004^{120}-1)} \times 0.004$$

$$= \$735.634$$

/2

Marker's Comments

MINUS 1 MARK FOR MAJOR MISTAK

/2

$$A_n = 1.004^n \times 70000 - 450(1 + 1.004 + 1.004^2 + \dots - 1.004^{n-1})$$

$$0 = \quad " \quad - 450\left[\frac{1(1.004^n - 1)}{1.004-1}\right)$$

$$= \quad " \quad - 450 \times 250(1.004^n - 1)$$

$$0 = 1.004^n[70000 - 250 \times 450) + 250 \times 450$$

$$1004^n = \frac{45}{17}$$

$$n \ln 1.004 = \ln 45/17$$

$$n = 243.148$$

= 20 yrs 4 month

/2

LOSING 1 MARK FOR NOT TRANSFERIN TO MONTHS

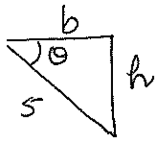
/2

(i) Area of the trapezium

$= 2 \times$ area of one triangle + area of the rectangle

$= 2 \times \frac{1}{2} \times \text{base} \times \text{height} + 5 \times 5\sin\theta$

(as .



, $\cos\theta = \frac{b}{5}$

$\Rightarrow b = 5\cos\theta$

$\sin\theta = \frac{h}{5} \Rightarrow h = 5\sin\theta$)

good

$= 5\cos\theta \times 5\sin\theta + 25\sin\theta$

$= 25(\sin\theta + \sin\theta\cos\theta)$

(ii) $\frac{dA}{d\theta} = 25\left(\cos\theta + \sin\theta(-\sin\theta) + \cos\theta\cos\theta\right)$

$= 25(\cos\theta - \sin^2\theta + \cos^2\theta)$

$= 25\left(\cos\theta - (1-\cos^2\theta) + \cos^2\theta\right)$

$= 25(2\cos^2\theta + \cos\theta - 1)$

Some students could not convert $\sin^2\theta$ into $\cos^2\theta$

(iii) for max/min OR turning point,

$\frac{dA}{d\theta} = 0 \quad \Rightarrow \quad 2\cos^2\theta + \cos\theta - 1 = 0$

$\cos\theta = \frac{-1 \pm \sqrt{1+8}}{4}$

$= \frac{-1 \pm 3}{4}, \quad -1, \frac{1}{2}$

$\theta = \pi$ or $\pi/3$

Since $\theta \leq \pi/2 \quad \therefore \theta = \pi/3$ is the only option

few students forgot to write a reason why they did not use $\theta = \pi$

Exam MATHEMATICS : Question……

Suggested Solutions	Marker's Comments
(iv) $A = 25\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}\right)$ $\quad \cos \pi/3 = \frac{1}{2}$ $\quad \sin \pi/3 = \frac{\sqrt{3}}{2}$ $= 32$ (nearest square metre) some students gave the answer in surd form	